SAR Assimilation for Near-Shore Spectral Wave Models

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LONG-TERM GOALS

The long term goal of this effort is to develop a methodology to estimate bathymetry and wave-energy dissipation from satellite-based SAR imagery.

OBJECTIVES

This program will attempt to develop a capability for estimating bathymetry via assimilation of SAR imagery into a near-shore spectral wave model. A capability for estimating dissipation of wave energy due to bottom friction will also be developed.

APPROACH

The procedure being developed makes use of a variational data assimilation capability for the SWAN near-shore spectral wave model of Booij *et al.* (1999). The bathymetry estimation procedure makes use of the SWAN wave-action balance model and a nonlinear model for the SAR image spectrum, along with adjoint forms of both models, to determine the bathymetry which minimizes the error between the estimated and observed SAR-image spectrum. A similar approach will be used to estimate wave energy dissipation due to bottom friction.

WORK COMPLETED

The work completed during the six months since the inception of the program includes development of the mathematical framework for the assimilation procedure and improvements in the convergence properties of the basic SAR assimilation procedure.

RESULTS

The expression for the predicted SAR-image spectrum derived by Hasselmann and Hasselmann (1991) and Krogstad (1992) can be written in a slightly modified form as

$$\hat{S}_i(k_x,k_y)=\frac{1}{(2\pi)^2}\iint G(x,y,k_x)e^{-i(k_x x+k_y y)}dxdy\ ,$$

where

$$G(x,y,k_x)=\left\{\rho_{rr}(x,y)+f(x,y,k_x)f(-x,-y,-k_x)\right\}e^{-k_x^2 C(x,y)}$$
$$f(x,y,k_x)=1+ik_x[\rho_{rv}(x,y)-\rho_{rv}(0,0)],\ \ C(x,y)=\rho_{vv}(0,0)-\rho_{vv}(x,y)$$
$$\rho_{rr}(x,y)=\mathrm{Re}\iint\left|T_r(k_x,k_y)\right|^2 S(k_x,k_y)e^{i(k_x x+k_y y)}dk_x dk_y$$
$$\rho_{rv}(x,y)=\mathrm{Re}\iint T_r(k_x,k_y)T_v^*(k_x,k_y)S(k_x,k_y)e^{i(k_x x+k_y y)}dk_x dk_y$$
$$\rho_{vv}(x,y)=\mathrm{Re}\iint\left|T_v(k_x,k_y)\right|^2 S(k_x,k_y)e^{i(k_x x+k_y y)}dk_x dk_y$$
$$T_r(k_x,k_y)=k\,m(k_x,k_y)\ \text{and}\ T_v(k_x,k_y)=\frac{R}{V\sigma}\left(gk_y\sin\theta-i\sigma^2\cos\theta\right).$$

Here $S(k_x, k_y)$ is the wave height spectrum, x and y are the azimuth and range coordinates and k_x and k_y are the corresponding wavenumbers, σ is the wave frequency, g is the gravitational acceleration, R is the range distance, V is the SAR platform velocity, and θ is the incidence angle. The function $m(k_x, k_y)$ is the radar modulation transfer function (mtf), which describes the variations in the radar backscatter due to surface tilt and hydrodynamic effects.

The above SAR model along with the SWAN model form the basis for the assimilation procedure used in this study. The methodology proceeds by inputting the error in the predicted SAR spectrum into the adjoint SAR model (Lyzenga 2002) whose output is input to the adjoint SWAN model. The resulting adjoint wave spectrum can be used in a procedure to minimize the error between the predicted and observed SAR image spectrum. Previous implementations of this assimilation procedure suffered from poor convergence properties. These difficulties have been traced to inaccuracies in the interpolation procedures used to go between the polar spectrum produced by the SWAN model, and the rectangular SAR spectrum. These errors would cause the procedure to sometimes diverge, due primarily to lack of energy conservation in the polar to rectangular conversion. The improved algorithm now converges reliably.

The desired bathymetry estimation algorithm is presently being implemented. Estimation of bathymetry via an assimilation approach requires a first guess for the bathymetry and, at each iteration of the assimilation loop, a bathymetry correction is made. The depth-dependent terms in the SWAN model form the basis for the gradient of the error in the estimated SAR spectrum, J, with respect to the bathymetry

$$\frac{\partial J}{\partial h}\propto\int N\left(\frac{\partial A}{\partial x}\frac{\delta C_x}{\delta h}+\frac{\partial A}{\partial y}\frac{\delta C_y}{\delta h}+\frac{\partial A}{\partial\sigma}\frac{\delta C_\sigma}{\delta h}+\frac{\partial A}{\partial\theta}\frac{\delta C_\theta}{\delta h}\right)d\mathbf{s}+\int\frac{A}{\sigma}\frac{\delta S}{\delta h}d\mathbf{s}\,,$$

where $N(\mathbf{x},\mathbf{s})$ is the action spectral density, $A(\mathbf{x},\mathbf{s})$ is the adjoint action spectral density, and the integrals are over spectral space. Here, the $\delta C/\delta h$ terms represent the first variation of the given propagation velocity with respect to the water depth $h(\mathbf{x})$, calculated from the expressions for the propagation velocities given above, and the $\delta S/\delta h$ is the first variation of the source term with respect the water depth. Using this expression, a spatially-varying bathymetry correction can be calculated from a combination of the predicted wave-action spectrum and the adjoint wave action spectrum, obtained from the SWAN model and its adjoint.

IMPACT/APPLICATIONS

This represents a unique approach to bathymetry estimation. Other approaches rely primarily on wave kinematics, and use the dispersion relation to determine the water depth for the observed wavelength and phase speed. In the above-described approach, the variations with water depth of wave energy flux, both in spectral and physical space, will provide the bathymetry estimate. Hence, the spatial variations in the wave energy, including the effects of refraction, will be used, rather than wave kinematics. This approach does involve some risks, since it depends on the accuracy with which spectral and spatial wave energy distribution can be calculated using the assimilation process and, therefore, to some extent on the accuracy of the modeling of the source terms in the SWAN model. However, the returns could be great if accurate bathymetry estimates can be obtained over an entire 100 km x 100 km area from a single ERS SAR image, or a small number of images.

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